Maple 2018.2 Integration Test Results
on the problems in "4 Trig functions/4.6 Cosecant"

Test results for the 23 problems in "4.6.0 (a csc)^m (b trg)^n.txt"

Problem 7: Result more than twice size of optimal antiderivative.

$$\int (c \csc(b x + a))^{7/2} \, dx$$

Optimal(type 4, 119 leaves, 4 steps):

$$-\frac{2 c \cos(b x + a) (c \csc(b x + a))^{5/2}}{5 b} - \frac{6 c^3 \cos(b x + a) \sqrt{c \csc(b x + a)}}{5 b} + \frac{6 c^4 \sqrt{\sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b x}{2}\right)^2}\, \mathrm{EllipticE}\left(\cos\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b x}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{a}{2} + \frac{\pi}{4} + \frac{b x}{2}\right) b \sqrt{c \csc(b x + a)}\, \sqrt{\sin(b x + a)}}$$

Result(type 4, 1084 leaves):

$$\frac{1}{5 b} \Bigg(\sqrt{2} \Bigg($$

$$-6 \sqrt{\frac{-I (I \sin(b x + a) - \cos(b x + a) + 1)}{\sin(b x + a)}} \sqrt{\frac{-I (I \sin(b x + a) + \cos(b x + a) - 1)}{\sin(b x + a)}} \sqrt{\frac{-I (-1 + \cos(b x + a))}{\sin(b x + a)}}\, \mathrm{EllipticE}\Bigg($$

$$\sqrt{\frac{-I (I \sin(b x + a) - \cos(b x + a) + 1)}{\sin(b x + a)}}, \frac{\sqrt{2}}{2}\Bigg) \cos(b x + a)^3$$

$$+ 3 \sqrt{\frac{-I (I \sin(b x + a) - \cos(b x + a) + 1)}{\sin(b x + a)}} \sqrt{\frac{-I (I \sin(b x + a) + \cos(b x + a) - 1)}{\sin(b x + a)}} \sqrt{\frac{-I (-1 + \cos(b x + a))}{\sin(b x + a)}}\, \mathrm{EllipticF}\Bigg($$

$$\sqrt{\frac{-I (I \sin(b x + a) - \cos(b x + a) + 1)}{\sin(b x + a)}}, \frac{\sqrt{2}}{2}\Bigg) \cos(b x + a)^3$$

$$- 6 \sqrt{\frac{-I (I \sin(b x + a) - \cos(b x + a) + 1)}{\sin(b x + a)}} \sqrt{\frac{-I (I \sin(b x + a) + \cos(b x + a) - 1)}{\sin(b x + a)}} \sqrt{\frac{-I (-1 + \cos(b x + a))}{\sin(b x + a)}}\, \mathrm{EllipticE}\Bigg($$

$$\sqrt{\frac{-I (I \sin(b x + a) - \cos(b x + a) + 1)}{\sin(b x + a)}}, \frac{\sqrt{2}}{2}\Bigg) \cos(b x + a)^2$$

$$+ 3 \sqrt{\frac{-I (I \sin(b x + a) - \cos(b x + a) + 1)}{\sin(b x + a)}} \sqrt{\frac{-I (I \sin(b x + a) + \cos(b x + a) - 1)}{\sin(b x + a)}} \sqrt{\frac{-I (-1 + \cos(b x + a))}{\sin(b x + a)}}\, \mathrm{EllipticF}\Bigg($$

$$\sqrt{\frac{-I (I \sin(b x + a) - \cos(b x + a) + 1)}{\sin(b x + a)}}, \frac{\sqrt{2}}{2}\Bigg) \cos(b x + a)^2$$

$$+ 6 \sqrt{\frac{-I (I \sin(b x + a) - \cos(b x + a) + 1)}{\sin(b x + a)}} \sqrt{\frac{-I (I \sin(b x + a) + \cos(b x + a) - 1)}{\sin(b x + a)}} \sqrt{\frac{-I (-1 + \cos(b x + a))}{\sin(b x + a)}}\, \mathrm{EllipticE}\Bigg($$

$$\sqrt{\frac{-I (I \sin(b x + a) - \cos(b x + a) + 1)}{\sin(b x + a)}}, \frac{\sqrt{2}}{2}\Bigg) \cos(b x + a)$$

$$-3\sqrt{\frac{-I\,(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}}\sqrt{\frac{-I\,(I\sin(bx+a)+\cos(bx+a)-1)}{\sin(bx+a)}}\sqrt{\frac{-I\,(-1+\cos(bx+a))}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\cos(bx+a)$$
$$+6\sqrt{\frac{-I\,(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}}\sqrt{\frac{-I\,(I\sin(bx+a)+\cos(bx+a)-1)}{\sin(bx+a)}}\sqrt{\frac{-I\,(-1+\cos(bx+a))}{\sin(bx+a)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{-I\,(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)$$
$$-3\sqrt{\frac{-I\,(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}}\sqrt{\frac{-I\,(I\sin(bx+a)+\cos(bx+a)-1)}{\sin(bx+a)}}\sqrt{\frac{-I\,(-1+\cos(bx+a))}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)+3\sqrt{2}\cos(bx+a)^2-\sqrt{2}\cos(bx+a)-3\sqrt{2}\Bigg)\left(\frac{c}{\sin(bx+a)}\right)^{7/2}\sin(bx+a)\Bigg)$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int (c\csc(bx+a))^{5/2}\,dx$$

Optimal(type 4, 95 leaves, 3 steps):

$$-\frac{2c\cos(bx+a)\,(c\csc(bx+a))^{3/2}}{3b}-\frac{2c^2\sqrt{\sin\left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right)^2}\,\mathrm{EllipticF}\left(\cos\left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right),\sqrt{2}\right)\sqrt{c\csc(bx+a)}\sqrt{\sin(bx+a)}}{3\sin\left(\frac{a}{2}+\frac{\pi}{4}+\frac{bx}{2}\right)b}$$

Result(type 4, 326 leaves):

$$\frac{1}{3b\sin(bx+a)^3}\Bigg(\sqrt{2}\,(-1+\cos(bx+a))^2\left(I\sqrt{\frac{-I\,(-1+\cos(bx+a))}{\sin(bx+a)}}\sqrt{\frac{-I\,(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}}\sqrt{\frac{-I\,(I\sin(bx+a)+\cos(bx+a)-1)}{\sin(bx+a)}}\sin(bx+a)\,\mathrm{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\cos(bx+a)\right.$$
$$\left.+I\sqrt{\frac{-I\,(-1+\cos(bx+a))}{\sin(bx+a)}}\sqrt{\frac{-I\,(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}}\sqrt{\frac{-I\,(I\sin(bx+a)+\cos(bx+a)-1)}{\sin(bx+a)}}\sin(bx+a)\,\mathrm{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)-\sqrt{2}\cos(bx+a)\right)(\cos(bx+a)+1)^2\left(\frac{c}{\sin(bx+a)}\right)^{5/2}\Bigg)$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int (c\csc(bx+a))^{3/2}\,dx$$

Optimal(type 4, 95 leaves, 3 steps):

$$-\frac{2 c \cos(b x+a) \sqrt{c \csc(b x+a)}}{b}+\frac{2 c^{2} \sqrt{\sin\left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right)^{2}}\, \operatorname{EllipticE}\left(\cos\left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right) b \sqrt{c \csc(b x+a)}\, \sqrt{\sin(b x+a)}}$$

Result(type 4, 532 leaves):

$$-\frac{1}{b}\Bigg(\sqrt{2}\Bigg(-2\sqrt{\frac{-I(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}}\sqrt{\frac{-I(I\sin(bx+a)+\cos(bx+a)-1)}{\sin(bx+a)}}\sqrt{\frac{-I(-1+\cos(bx+a))}{\sin(bx+a)}}\operatorname{EllipticE}\left(\sqrt{\frac{-I(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\cos(bx+a)$$

$$+\sqrt{\frac{-I(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}}\sqrt{\frac{-I(I\sin(bx+a)+\cos(bx+a)-1)}{\sin(bx+a)}}\sqrt{\frac{-I(-1+\cos(bx+a))}{\sin(bx+a)}}\operatorname{EllipticF}\left(\sqrt{\frac{-I(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\cos(bx+a)$$

$$-2\sqrt{\frac{-I(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}}\sqrt{\frac{-I(I\sin(bx+a)+\cos(bx+a)-1)}{\sin(bx+a)}}\sqrt{\frac{-I(-1+\cos(bx+a))}{\sin(bx+a)}}\operatorname{EllipticE}\left(\sqrt{\frac{-I(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)$$

$$+\sqrt{\frac{-I(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}}\sqrt{\frac{-I(I\sin(bx+a)+\cos(bx+a)-1)}{\sin(bx+a)}}\sqrt{\frac{-I(-1+\cos(bx+a))}{\sin(bx+a)}}\operatorname{EllipticF}\left(\sqrt{\frac{-I(I\sin(bx+a)-\cos(bx+a)+1)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)+\sqrt{2}\Bigg)\left(\frac{c}{\sin(bx+a)}\right)^{3/2}\sin(bx+a)\Bigg)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(c \csc(b x+a))^{5/2}}\, dx$$

Optimal(type 4, 97 leaves, 3 steps):

$$-\frac{2 \cos(b x+a)}{5 b c (c \csc(b x+a))^{3/2}}-\frac{6 \sqrt{\sin\left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right)^{2}}\, \operatorname{EllipticE}\left(\cos\left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right), \sqrt{2}\right)}{5 \sin\left(\frac{a}{2}+\frac{\pi}{4}+\frac{b x}{2}\right) b c^{2} \sqrt{c \csc(b x+a)}\, \sqrt{\sin(b x+a)}}$$

Result(type 4, 562 leaves):

$$-\frac{1}{5 b \left(\frac{c}{\sin(b x+a)}\right)^{5/2} \sin(b x+a)^3}\left(\sqrt{2}\left(6 \sqrt{\frac{-I\,(I\sin(b x+a)-\cos(b x+a)+1)}{\sin(b x+a)}}\sqrt{\frac{-I\,(I\sin(b x+a)+\cos(b x+a)-1)}{\sin(b x+a)}}\sqrt{\frac{-I\,(-1+\cos(b x+a))}{\sin(b x+a)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{-I\,(I\sin(b x+a)-\cos(b x+a)+1)}{\sin(b x+a)}}, \frac{\sqrt{2}}{2}\right)\cos(b x+a)\right.\right.$$
$$-3\sqrt{\frac{-I\,(I\sin(b x+a)-\cos(b x+a)+1)}{\sin(b x+a)}}\sqrt{\frac{-I\,(I\sin(b x+a)+\cos(b x+a)-1)}{\sin(b x+a)}}\sqrt{\frac{-I\,(-1+\cos(b x+a))}{\sin(b x+a)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(b x+a)-\cos(b x+a)+1)}{\sin(b x+a)}}, \frac{\sqrt{2}}{2}\right)\cos(b x+a)$$
$$+6\sqrt{\frac{-I\,(I\sin(b x+a)-\cos(b x+a)+1)}{\sin(b x+a)}}\sqrt{\frac{-I\,(I\sin(b x+a)+\cos(b x+a)-1)}{\sin(b x+a)}}\sqrt{\frac{-I\,(-1+\cos(b x+a))}{\sin(b x+a)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{-I\,(I\sin(b x+a)-\cos(b x+a)+1)}{\sin(b x+a)}}, \frac{\sqrt{2}}{2}\right)$$
$$\left.\left.-3\sqrt{\frac{-I\,(I\sin(b x+a)-\cos(b x+a)+1)}{\sin(b x+a)}}\sqrt{\frac{-I\,(I\sin(b x+a)+\cos(b x+a)-1)}{\sin(b x+a)}}\sqrt{\frac{-I\,(-1+\cos(b x+a))}{\sin(b x+a)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(b x+a)-\cos(b x+a)+1)}{\sin(b x+a)}}, \frac{\sqrt{2}}{2}\right)-\sqrt{2}\cos(b x+a)^3+4\sqrt{2}\cos(b x+a)-3\sqrt{2}\right)\right)$$

Problem 11: Unable to integrate problem.

$$\int \frac{1}{\csc(b x+a)^{2/3}}\, dx$$

Optimal(type 5, 43 leaves, 2 steps):

$$\frac{3\cos(b x+a)\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{6}\right], \left[\frac{11}{6}\right], \sin(b x+a)^2\right)}{5 b \csc(b x+a)^{5/3}\sqrt{\cos(b x+a)^2}}$$

Result(type 8, 148 leaves):

$$-\frac{3\,I\,2^{1/3}}{4 b\left(\frac{I\,e^{I(b x+a)}}{(e^{I(b x+a)})^2-1}\right)^{2/3}}+\frac{\left(\int -\frac{2}{\left(-(e^{I(b x+a)})^2\left((e^{I(b x+a)})^2-1\right)\right)^{1/3}}\, dx\right)2^{1/3}\left(-(e^{I(b x+a)})^2\left((e^{I(b x+a)})^2-1\right)\right)^{1/3}}{2\left(\frac{I\,e^{I(b x+a)}}{(e^{I(b x+a)})^2-1}\right)^{2/3}\left((e^{I(b x+a)})^2-1\right)}$$

Problem 12: Unable to integrate problem.

$$\int \frac{1}{(c\csc(b x+a))^{2/3}}\, dx$$

Optimal(type 5, 46 leaves, 2 steps):

$$\frac{3\, c \cos(b x + a)\, \mathrm{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{6}\right], \left[\frac{11}{6}\right], \sin(b x + a)^2\right)}{5\, b\, (c \csc(b x + a))^{5/3} \sqrt{\cos(b x + a)^2}}$$

Result(type 8, 156 leaves):

$$-\frac{3\, I\, 2^{1/3}}{4\, b \left(\frac{I\, c\, e^{I (b x + a)}}{(e^{I (b x + a)})^2 - 1}\right)^{2/3}} + \frac{\left(\int -\frac{2}{\left(-c^2 (e^{I (b x + a)})^2 \left((e^{I (b x + a)})^2 - 1\right)\right)^{1/3}} \, dx\right) 2^{1/3} \left(-c^2 (e^{I (b x + a)})^2 \left((e^{I (b x + a)})^2 - 1\right)\right)^{1/3}}{2 \left(\frac{I\, c\, e^{I (b x + a)}}{(e^{I (b x + a)})^2 - 1}\right)^{2/3} \left((e^{I (b x + a)})^2 - 1\right)}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int (\csc(x)^2)^{7/2} \, dx$$

Optimal(type 3, 36 leaves, 5 steps):

$$-\frac{5\, \mathrm{arcsinh}(\cot(x))}{16} - \frac{5 \cot(x)\, (\csc(x)^2)^{3/2}}{24} - \frac{\cot(x)\, (\csc(x)^2)^{5/2}}{6} - \frac{5 \cot(x) \sqrt{\csc(x)^2}}{16}$$

Result(type 3, 100 leaves):

$$-\frac{1}{96}\left(\sqrt{4}\left(15 \cos(x)^6 \ln\left(-\frac{-1 + \cos(x)}{\sin(x)}\right) + 15 \cos(x)^5 - 45 \cos(x)^4 \ln\left(-\frac{-1 + \cos(x)}{\sin(x)}\right) - 40 \cos(x)^3 + 45 \cos(x)^2 \ln\left(-\frac{-1 + \cos(x)}{\sin(x)}\right) + 33 \cos(x) - 15 \ln\left(-\frac{-1 + \cos(x)}{\sin(x)}\right)\right) \sin(x) \left(-\frac{1}{\cos(x)^2 - 1}\right)^{7/2}\right)$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int (\csc(x)^2)^{3/2} \, dx$$

Optimal(type 3, 16 leaves, 3 steps):

$$-\frac{\mathrm{arcsinh}(\cot(x))}{2} - \frac{\cot(x) \sqrt{\csc(x)^2}}{2}$$

Result(type 3, 51 leaves):

$$-\frac{\sqrt{4}\left(\cos(x)^2 \ln\left(-\frac{-1 + \cos(x)}{\sin(x)}\right) + \cos(x) - \ln\left(-\frac{-1 + \cos(x)}{\sin(x)}\right)\right) \sin(x) \left(-\frac{1}{\cos(x)^2 - 1}\right)^{3/2}}{4}$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int (a \csc(x)^3)^{3/2} \, dx$$

Optimal(type 4, 78 leaves, 5 steps):

$$-\frac{10\,a\cos(x)\sqrt{a\csc(x)^3}}{21}-\frac{2\,a\cot(x)\csc(x)\sqrt{a\csc(x)^3}}{7}-\frac{10\,a\sqrt{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)^2}\,\text{EllipticF}\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right),\sqrt{2}\right)\sin(x)^{3/2}\sqrt{a\csc(x)^3}}{21\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$

Result(type 4, 399 leaves):

$$-\frac{1}{168\sin(x)^3}\Bigg(\sqrt{8}\,(\cos(x)+1)^2(-1+\cos(x))^2\Bigg(5\,I\sqrt{2}\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}}\sqrt{\frac{-I\,(I\sin(x)+\cos(x)-1)}{\sin(x)}}\sqrt{\frac{-I\,(-1+\cos(x))}{\sin(x)}}\,\text{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}},\frac{\sqrt{2}}{2}\right)\cos(x)^3\sin(x)$$
$$+5\,I\sqrt{2}\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}}\sqrt{\frac{-I\,(I\sin(x)+\cos(x)-1)}{\sin(x)}}\sqrt{\frac{-I\,(-1+\cos(x))}{\sin(x)}}\,\text{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}},\frac{\sqrt{2}}{2}\right)\cos(x)^2\sin(x)$$
$$-5\,I\sqrt{2}\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}}\sqrt{\frac{-I\,(I\sin(x)+\cos(x)-1)}{\sin(x)}}\sqrt{\frac{-I\,(-1+\cos(x))}{\sin(x)}}\,\text{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}},\frac{\sqrt{2}}{2}\right)\cos(x)\sin(x)$$
$$-5\,I\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}}\sqrt{2}\sqrt{\frac{-I\,(I\sin(x)+\cos(x)-1)}{\sin(x)}}\sqrt{\frac{-I\,(-1+\cos(x))}{\sin(x)}}\,\text{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}},\frac{\sqrt{2}}{2}\right)\sin(x)-10\cos(x)^3+16\cos(x)\Bigg)\left(-\frac{2\,a}{\sin(x)\left(\cos(x)^2-1\right)}\right)^{3/2}\Bigg)$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(a\csc(x)^3)^{3/2}}\,dx$$

Optimal(type 4, 86 leaves, 5 steps):

$$-\frac{14\cos(x)}{45\,a\sqrt{a\csc(x)^3}}-\frac{14\sqrt{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)^2}\,\text{EllipticE}\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right),\sqrt{2}\right)}{15\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)a\sin(x)^{3/2}\sqrt{a\csc(x)^3}}-\frac{2\cos(x)\sin(x)^2}{9\,a\sqrt{a\csc(x)^3}}$$

Result(type 4, 376 leaves):

$$-\frac{1}{45\left(-\frac{2a}{\sin(x)\left(\cos(x)^2-1\right)}\right)^{3/2}\sin(x)^5}\left(\sqrt{8}\left(42\sqrt{\frac{-I\,(I\sin(x)+\cos(x)-1)}{\sin(x)}}\sqrt{2}\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}}\sqrt{\frac{-I\,(-1+\cos(x))}{\sin(x)}}\right.\right.$$

$$\mathrm{EllipticE}\left(\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}},\frac{\sqrt{2}}{2}\right)\cos(x)$$

$$-21\sqrt{\frac{-I\,(I\sin(x)+\cos(x)-1)}{\sin(x)}}\sqrt{2}\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}}\sqrt{\frac{-I\,(-1+\cos(x))}{\sin(x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}},\right.$$

$$\left.\frac{\sqrt{2}}{2}\right)\cos(x)$$

$$+42\sqrt{\frac{-I\,(I\sin(x)+\cos(x)-1)}{\sin(x)}}\sqrt{2}\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}}\sqrt{\frac{-I\,(-1+\cos(x))}{\sin(x)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}},\frac{\sqrt{2}}{2}\right)$$

$$-21\sqrt{\frac{-I\,(I\sin(x)+\cos(x)-1)}{\sin(x)}}\sqrt{2}\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}}\sqrt{\frac{-I\,(-1+\cos(x))}{\sin(x)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-I\,(I\sin(x)-\cos(x)+1)}{\sin(x)}},\frac{\sqrt{2}}{2}\right)$$

$$\left.\left.+10\cos(x)^5-34\cos(x)^3+66\cos(x)-42\right)\right)$$

Problem 23: Unable to integrate problem.

$$\int (a\csc(fx+e))^m (b\csc(fx+e))^n\,dx$$

Optimal(type 5, 81 leaves, 3 steps):

$$\frac{a\cos(fx+e)\,(a\csc(fx+e))^{-1+m}(b\csc(fx+e))^n\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}-\frac{m}{2}-\frac{n}{2}\right],\left[\frac{3}{2}-\frac{m}{2}-\frac{n}{2}\right],\sin(fx+e)^2\right)}{f(1-m-n)\sqrt{\cos(fx+e)^2}}$$

Result(type 8, 23 leaves):

$$\int (a\csc(fx+e))^m (b\csc(fx+e))^n\,dx$$

Test results for the 19 problems in "4.6.1.2 (d csc)^n (a+b csc)^m.txt"

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{\sin(x)^4}{a+a\csc(x)}\,dx$$

Optimal(type 3, 58 leaves, 7 steps):

$$\frac{15x}{8a}+\frac{4\cos(x)}{a}-\frac{4\cos(x)^3}{3a}-\frac{15\cos(x)\sin(x)}{8a}-\frac{5\cos(x)\sin(x)^3}{4a}+\frac{\cos(x)\sin(x)^3}{a+a\csc(x)}$$

Result(type 3, 184 leaves):

$$\frac{7\tan\left(\frac{x}{2}\right)^{7}}{4a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}+\frac{2\tan\left(\frac{x}{2}\right)^{6}}{a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}+\frac{15\tan\left(\frac{x}{2}\right)^{5}}{4a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}+\frac{10\tan\left(\frac{x}{2}\right)^{4}}{a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}-\frac{15\tan\left(\frac{x}{2}\right)^{3}}{4a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}+\frac{34\tan\left(\frac{x}{2}\right)^{2}}{3a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}$$
$$-\frac{7\tan\left(\frac{x}{2}\right)}{4a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}+\frac{10}{3a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}+\frac{15\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{4a}+\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

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Problem 5: Result more than twice size of optimal antiderivative.
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$$\int\sqrt{a+a\csc(x)}\,dx$$

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Optimal(type 3, 20 leaves, 2 steps):
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$$-2\arctan\left(\frac{\cot(x)\sqrt{a}}{\sqrt{a+a\csc(x)}}\right)\sqrt{a}$$

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Result(type 3, 198 leaves):
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$$-\frac{1}{2(-1+\cos(x)-\sin(x))}\Bigg(\sqrt{2}\sqrt{\frac{a(1+\sin(x))}{\sin(x)}}\sin(x)\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\left(\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}\right)\right.$$
$$\left.+4\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}+1\right)+4\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}-1\right)+\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}\right)\right)\Bigg)$$

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Problem 6: Result more than twice size of optimal antiderivative.
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$$\int\frac{1}{\sqrt{a+a\csc(x)}}\,dx$$

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Optimal(type 3, 47 leaves, 5 steps):
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$$-\frac{2\arctan\left(\frac{\cot(x)\sqrt{a}}{\sqrt{a+a\csc(x)}}\right)}{\sqrt{a}}+\frac{\arctan\left(\frac{\cot(x)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\csc(x)}}\right)\sqrt{2}}{\sqrt{a}}$$

```
Result(type 3, 220 leaves):
```

$$\frac{1}{4\sqrt{\frac{a(1+\sin(x))}{\sin(x)}}\,\sin(x)\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}}\Bigg(\sqrt{2}\,(-1+\cos(x)-\sin(x))\Bigg(4\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)-\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}\Bigg)-4\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}+1\right)-4\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}-1\right)-\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}\Bigg)\Bigg)\Bigg)$$

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Problem 7: Result more than twice size of optimal antiderivative.
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$$\int\frac{1}{(a+a\csc(x))^{3/2}}\,dx$$

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Optimal(type 3, 60 leaves, 6 steps):
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$$-\frac{2\arctan\left(\frac{\cot(x)\sqrt{a}}{\sqrt{a+a\csc(x)}}\right)}{a^{3/2}}+\frac{\cot(x)}{2\,(a+a\csc(x))^{3/2}}+\frac{5\arctan\left(\frac{\cot(x)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\csc(x)}}\right)\sqrt{2}}{4\,a^{3/2}}$$

```
Result(type 3, 1140 leaves):
```

$$-\frac{1}{8\left(\frac{a(1+\sin(x))}{\sin(x)}\right)^{3/2}\sin(x)^3\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{3/2}}\Bigg(\sqrt{2}\,(-1+\cos(x))\Bigg(10\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)\cos(x)\sin(x)+\sqrt{2}\cos(x)^2\Bigg($$

$$-\frac{-1+\cos(x)}{\sin(x)}\Bigg)^{3/2}-\sqrt{2}\sin(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{3/2}-\sqrt{2}\cos(x)^2\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}-2\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}\Bigg)\cos(x)\sin(x)-8\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}+1\right)\cos(x)\sin(x)$$

$$- 8 \arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}-1\right)\cos(x)\sin(x) - 2\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}\right)\cos(x)\sin(x)$$

$$+ 10\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)\cos(x)^2 + 10\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)\cos(x) - 20\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)\sin(x) - \sqrt{2}\,\Bigg($$

$$-\frac{-1+\cos(x)}{\sin(x)}\Bigg)^{3/2} - 2\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}\right)\cos(x)^2 - 8\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}+1\right)\cos(x)^2$$

$$- 8\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}-1\right)\cos(x)^2 - 2\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}\right)\cos(x)^2 - 2\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}\Bigg)\cos(x) + 4\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}\right)\sin(x)$$

$$- 8\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}+1\right)\cos(x) + 16\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}+1\right)\sin(x) - 8\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}-1\right)\cos(x)$$

$$+ 16\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}-1\right)\sin(x) - 2\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}\right)\cos(x) + 4\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}\Bigg)\sin(x) + 16\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}-1\right) + 4\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}\Bigg) + \sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\sin(x) + \sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2} + 4\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}\Bigg)+16\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}+1\right)-20\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)$$

$$-\sqrt{2}\cos(x)\sin(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{3/2}+\sqrt{2}\cos(x)\sin(x)\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\Bigg)\Bigg)$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{(a+a\csc(x))^{5/2}}\,dx$$

Optimal(type 3, 75 leaves, 7 steps):

$$-\frac{2\arctan\left(\frac{\cot(x)\sqrt{a}}{\sqrt{a+a\csc(x)}}\right)}{a^{5/2}}+\frac{\cot(x)}{4\,(a+a\csc(x))^{5/2}}+\frac{11\cot(x)}{16\,a\,(a+a\csc(x))^{3/2}}+\frac{43\arctan\left(\frac{\cot(x)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\csc(x)}}\right)\sqrt{2}}{32\,a^{5/2}}$$

Result(type 3, 1960 leaves):

$$-\frac{1}{128\left(\frac{a\,(1+\sin(x))}{\sin(x)}\right)^{5/2}\sin(x)^5\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{5/2}}\Bigg(\sqrt{2}\,(-1+\cos(x))^2\Bigg(-344\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)\cos(x)\sin(x)+19\sqrt{2}\cos(x)^2\Bigg($$

$$-\frac{-1+\cos(x)}{\sin(x)}\Bigg)^{3/2}-19\sqrt{2}\sin(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{3/2}+11\sqrt{2}\cos(x)^2\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}+64\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}\Bigg)\cos(x)\sin(x)+256\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}+1\right)\cos(x)\sin(x)$$

$$+256\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}-1\right)\cos(x)\sin(x)+64\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}\right)\cos(x)\sin(x)$$

$$- 516\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)\cos(x)^{2} - 344\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)\cos(x) + 688\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)\sin(x) - 32\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) - \cos(x) + \sin(x) + 1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) + \cos(x) - \sin(x) - 1}\Bigg)\cos(x)^{3} - 128\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2} + 1\right)\cos(x)^{3} - 128\arctan\Bigg(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}$$

$$-1\Bigg)\cos(x)^{3} - 32\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) + \cos(x) - \sin(x) - 1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) - \cos(x) + \sin(x) + 1}\right)\cos(x)^{3} + 11\sqrt{2}\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{7/2} + 19\sqrt{2}\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{5/2}$$

$$- 19\sqrt{2}\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{3/2} + 96\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) - \cos(x) + \sin(x) + 1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) + \cos(x) - \sin(x) - 1}\right)\cos(x)^{2} + 384\arctan\Bigg(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}$$

$$+1\Bigg)\cos(x)^{2} + 384\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2} - 1\right)\cos(x)^{2} + 96\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) + \cos(x) - \sin(x) - 1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) - \cos(x) + \sin(x) + 1}\right)\cos(x)^{2} + 64\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) - \cos(x) + \sin(x) + 1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) + \cos(x) - \sin(x) - 1}\Bigg)\cos(x) - 128\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) - \cos(x) + \sin(x) + 1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) + \cos(x) - \sin(x) - 1}\right)\sin(x)$$

$$+ 256\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2} + 1\right)\cos(x) - 512\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2} + 1\right)\sin(x) + 256\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2} - 1\right)\cos(x)$$

$$- 512\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2} - 1\right)\sin(x) + 64\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) + \cos(x) - \sin(x) - 1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) - \cos(x) + \sin(x) + 1}\right)\cos(x) - 128\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) + \cos(x) - \sin(x) - 1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x) - \cos(x) + \sin(x) + 1}\Bigg)\sin(x) - 512\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2} - 1\right) - 128\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\,\sin(x)+\cos(x)-\sin(x)-1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\,\sin(x)-\cos(x)+\sin(x)+1}\Bigg)+38\sqrt{2}\cos(x)\sin(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{5/2}+11\sqrt{2}\cos(x)^{2}\sin(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{7/2}$$

$$+19\sqrt{2}\cos(x)^{2}\sin(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{5/2}-19\sqrt{2}\cos(x)^{2}\sin(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{3/2}-11\sqrt{2}\cos(x)^{2}\sin(x)\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}$$

$$-172\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)\cos(x)^{2}\sin(x)+22\sqrt{2}\cos(x)\sin(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{7/2}-11\sqrt{2}\cos(x)^{3}\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{7/2}$$

$$-11\sqrt{2}\cos(x)^{2}\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{7/2}-19\sqrt{2}\cos(x)^{3}\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{5/2}+11\sqrt{2}\cos(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{7/2}+11\sqrt{2}\sin(x)\Bigg($$

$$-\frac{-1+\cos(x)}{\sin(x)}\Bigg)^{7/2}-19\sqrt{2}\cos(x)^{2}\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{5/2}+19\sqrt{2}\cos(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{5/2}+19\sqrt{2}\sin(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{5/2}$$

$$+19\sqrt{2}\cos(x)^{3}\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{3/2}+11\sqrt{2}\cos(x)^{3}\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}-19\sqrt{2}\cos(x)\left(-\frac{-1+\cos(x)}{\sin(x)}\right)^{3/2}-11\sqrt{2}\cos(x)\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}$$

$$+32\ln\left(-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\,\sin(x)-\cos(x)+\sin(x)+1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\,\sin(x)+\cos(x)-\sin(x)-1}\right)\cos(x)^{2}\sin(x)+128\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}+1\right)\cos(x)^{2}\sin(x)$$

$$+128\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}-1\right)\cos(x)^{2}\sin(x)-11\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\,\sin(x)-11\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}-128\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\,\sin(x)-\cos(x)+\sin(x)+1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}\,\sin(x)+\cos(x)-\sin(x)-1}\Bigg)-512\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\,\sqrt{2}+1\right)+688\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)+32\ln\Bigg($$

$$-\frac{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)+\cos(x)-\sin(x)-1}{\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\sqrt{2}\sin(x)-\cos(x)+\sin(x)+1}\Bigg)\cos(x)^2\sin(x)+172\sqrt{2}\arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right)\cos(x)^3-38\sqrt{2}\cos(x)\sin(x)\Bigg(-\frac{-1+\cos(x)}{\sin(x)}\Bigg)^{3/2}-22\sqrt{2}\cos(x)\sin(x)\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\Bigg)\Bigg)$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int\sqrt{\csc(fx+e)}\sqrt{a+a\csc(fx+e)}\,dx$$

Optimal (type 3, 31 leaves, 2 steps):

$$-\frac{2\operatorname{arcsinh}\left(\frac{\cot(fx+e)\sqrt{a}}{\sqrt{a+a\csc(fx+e)}}\right)\sqrt{a}}{f}$$

Result (type 3, 113 leaves):

$$-\frac{\sqrt{2}\sqrt{\frac{1}{\sin(fx+e)}}\,(-1+\cos(fx+e))\sqrt{\frac{a(\sin(fx+e)+1)}{\sin(fx+e)}}\left(\operatorname{arcsinh}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{1}{\cos(fx+e)+1}}}\right)\right)}{f(-1+\cos(fx+e)-\sin(fx+e))\sqrt{\frac{1}{\cos(fx+e)+1}}}$$

Problem 10: Unable to integrate problem.

$$\int\frac{\sqrt{a+a\csc(dx+c)}}{\csc(dx+c)^{2/3}}\,dx$$

Optimal (type 4, 213 leaves, 4 steps):

$$-\frac{3a\cos(dx+c)\csc(dx+c)^{1/3}}{2d\sqrt{a+a\csc(dx+c)}}$$
$$-\left(3^{3/4}a^2\cot(dx+c)\left(1-\csc(dx+c)^{1/3}\right)\operatorname{EllipticF}\left(\frac{1-\csc(dx+c)^{1/3}-\sqrt{3}}{1-\csc(dx+c)^{1/3}+\sqrt{3}},\mathrm{I}\sqrt{3}+2\mathrm{I}\right)\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{\frac{1+\csc(dx+c)^{1/3}+\csc(dx+c)^{2/3}}{\left(1-\csc(dx+c)^{1/3}+\sqrt{3}\right)^2}}\right)\Bigg/\left(2d(a-a\csc(dx+c))\sqrt{a+a\csc(dx+c)}\sqrt{\frac{1-\csc(dx+c)^{1/3}}{\left(1-\csc(dx+c)^{1/3}+\sqrt{3}\right)^2}}\right)$$

Result (type 8, 402 leaves):

$$-\frac{3 I (e^{I(dx+c)}-2I)(e^{I(dx+c)}+I) 2^{1/3}\sqrt{\frac{a((e^{I(dx+c)})^2+2Ie^{I(dx+c)}-1)}{(e^{I(dx+c)})^2-1}}}{4d((e^{I(dx+c)})^2+2Ie^{I(dx+c)}-1)\left(\frac{Ie^{I(dx+c)}}{(e^{I(dx+c)})^2-1}\right)^{2/3}}+\left(\left(\int\frac{-1+\frac{Ie^{I(dx+c)}}{2}-\frac{(e^{I(dx+c)})^2}{2}}{\left(a^3((e^{I(dx+c)})^2+2Ie^{I(dx+c)}-1)^3((e^{I(dx+c)})^2-1)^5(e^{I(dx+c)})^4\right)^{1/6}}dx\right)\right.$$

$$\left.2^{1/3}\sqrt{\frac{a((e^{I(dx+c)})^2+2Ie^{I(dx+c)}-1)}{(e^{I(dx+c)})^2-1}}\left(a^3((e^{I(dx+c)})^2+2Ie^{I(dx+c)}-1)^3((e^{I(dx+c)})^2-1)^5(e^{I(dx+c)})^4\right)^{1/6}\right)\Bigg/\left(2((e^{I(dx+c)})^2\right.$$

$$\left.+2Ie^{I(dx+c)}-1)\left(\frac{Ie^{I(dx+c)}}{(e^{I(dx+c)})^2-1}\right)^{2/3}((e^{I(dx+c)})^2-1)\right)$$

Problem 11: Unable to integrate problem.

$$\int \csc(dx+c)^n\sqrt{a-a\csc(dx+c)}\,dx$$

Optimal(type 5, 65 leaves, 3 steps):

$$-\frac{2a\cos(dx+c)\csc(dx+c)^{1+n}\operatorname{hypergeom}\left(\left[\frac{1}{2},1-n\right],\left[\frac{3}{2}\right],1+\csc(dx+c)\right)}{d(-\csc(dx+c))^n\sqrt{a-a\csc(dx+c)}}$$

Result(type 8, 24 leaves):

$$\int \csc(dx+c)^n\sqrt{a-a\csc(dx+c)}\,dx$$

Problem 12: Unable to integrate problem.

$$\int (a+a\csc(fx+e))^m\sin(fx+e)^2\,dx$$

Optimal(type 6, 72 leaves, 3 steps):

$$-\frac{AppellF1\left(\frac{1}{2}+m,3,\frac{1}{2},\frac{3}{2}+m,1+\csc(fx+e),\frac{1}{2}+\frac{\csc(fx+e)}{2}\right)\cot(fx+e)(a+a\csc(fx+e))^m\sqrt{2}}{f(1+2m)\sqrt{1-\csc(fx+e)}}$$

Result(type 8, 23 leaves):

$$\int (a+a\csc(fx+e))^m\sin(fx+e)^2\,dx$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{\sin(x)^3}{a + b\csc(x)}\, dx$$

Optimal(type 3, 96 leaves, 8 steps):

$$-\frac{b\left(a^2+2b^2\right)x}{2a^4} - \frac{\left(2a^2+3b^2\right)\cos(x)}{3a^3} + \frac{b\cos(x)\sin(x)}{2a^2} - \frac{\cos(x)\sin(x)^2}{3a} - \frac{2b^4 \operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}}$$

Result(type 3, 212 leaves):

$$-\frac{b\tan\left(\frac{x}{2}\right)^5}{a^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3} - \frac{2b^2\tan\left(\frac{x}{2}\right)^4}{a^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3} - \frac{4\tan\left(\frac{x}{2}\right)^2}{a\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3} - \frac{4\tan\left(\frac{x}{2}\right)^2 b^2}{a^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3} + \frac{b\tan\left(\frac{x}{2}\right)}{a^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3} - \frac{4}{3a\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3}$$

$$-\frac{2b^2}{a^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3} - \frac{b\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^2} - \frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)b^3}{a^4} + \frac{2b^4\arctan\left(\frac{2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a^4\sqrt{-a^2+b^2}}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a + b\csc(dx+c)\right)^2}\, dx$$

Optimal(type 3, 103 leaves, 6 steps):

$$\frac{x}{a^2} + \frac{2b\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2} d} - \frac{b^2\cot(dx+c)}{a\left(a^2-b^2\right)d\left(a+b\csc(dx+c)\right)}$$

Result(type 3, 246 leaves):

$$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d a^2} - \frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b + 2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a + b\right)\left(a^2-b^2\right)}$$

$$-\frac{2 b^{2}}{d a \left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)^{2} b+2 \tan\left(\frac{d x}{2}+\frac{c}{2}\right) a+b\right) \left(a^{2}-b^{2}\right)}-\frac{4 b \arctan\left(\frac{2 b \tan\left(\frac{d x}{2}+\frac{c}{2}\right)+2 a}{2 \sqrt{-a^{2}+b^{2}}}\right)}{d \left(a^{2}-b^{2}\right) \sqrt{-a^{2}+b^{2}}}+\frac{2 b^{3} \arctan\left(\frac{2 b \tan\left(\frac{d x}{2}+\frac{c}{2}\right)+2 a}{2 \sqrt{-a^{2}+b^{2}}}\right)}{d a^{2} \left(a^{2}-b^{2}\right) \sqrt{-a^{2}+b^{2}}}$$

Problem 18: Unable to integrate problem.

$$\int \csc(fx+e) \left(a+b \csc(fx+e)\right)^{m} dx$$

Optimal(type 6, 90 leaves, 3 steps):

$$-\frac{AppellF1\left(\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, \frac{b\left(1-\csc(fx+e)\right)}{a+b}, \frac{1}{2}-\frac{\csc(fx+e)}{2}\right) \cot(fx+e) \left(a+b \csc(fx+e)\right)^{m} \sqrt{2}}{f\left(\frac{a+b \csc(fx+e)}{a+b}\right)^{m} \sqrt{1+\csc(fx+e)}}$$

Result(type 8, 21 leaves):

$$\int \csc(fx+e) \left(a+b \csc(fx+e)\right)^{m} dx$$

Test results for the 6 problems in "4.6.1.3 (d cos)^n (a+b csc)^m.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos(x)^{4}}{a+a \csc(x)} dx$$

Optimal(type 3, 36 leaves, 7 steps):

$$-\frac{x}{8 a}-\frac{\cos(x)^{3}}{3 a}-\frac{\cos(x) \sin(x)}{8 a}+\frac{\cos(x)^{3} \sin(x)}{4 a}$$

Result(type 3, 171 leaves):

$$-\frac{\tan\left(\frac{x}{2}\right)^{7}}{4 a \left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}-\frac{2 \tan\left(\frac{x}{2}\right)^{6}}{a \left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}+\frac{7 \tan\left(\frac{x}{2}\right)^{5}}{4 a \left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}-\frac{2 \tan\left(\frac{x}{2}\right)^{4}}{a \left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}-\frac{7 \tan\left(\frac{x}{2}\right)^{3}}{4 a \left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}-\frac{2 \tan\left(\frac{x}{2}\right)^{2}}{3 a \left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}$$
$$+\frac{\tan\left(\frac{x}{2}\right)}{4 a \left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}-\frac{2}{3 a \left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}}-\frac{\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{4 a}$$

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos(x)^2}{a + a\csc(x)}\, dx$$

Optimal(type 3, 23 leaves, 5 steps):

$$-\frac{x}{2a} - \frac{\cos(x)}{a} + \frac{\cos(x)\sin(x)}{2a}$$

Result(type 3, 86 leaves):

$$-\frac{\tan\left(\frac{x}{2}\right)^3}{a\left(\tan\left(\frac{x}{2}\right)^2+1\right)^2} - \frac{2\tan\left(\frac{x}{2}\right)^2}{a\left(\tan\left(\frac{x}{2}\right)^2+1\right)^2} + \frac{\tan\left(\frac{x}{2}\right)}{a\left(\tan\left(\frac{x}{2}\right)^2+1\right)^2} - \frac{2}{a\left(\tan\left(\frac{x}{2}\right)^2+1\right)^2} - \frac{\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{\sec(x)^4}{a + b\csc(x)}\, dx$$

Optimal(type 3, 99 leaves, 7 steps):

$$\frac{2a^3 b\,\mathrm{arctanh}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\sec(x)^3\,(b-a\sin(x))}{3\,(a^2-b^2)} - \frac{\sec(x)\,\left(3a^2 b - a\,(2a^2+b^2)\sin(x)\right)}{3\,(a^2-b^2)^2}$$

Result(type 3, 199 leaves):

$$-\frac{4}{3\left(\tan\left(\frac{x}{2}\right)-1\right)^3(4a+4b)} - \frac{2}{\left(\tan\left(\frac{x}{2}\right)-1\right)^2(4a+4b)} - \frac{a}{(a+b)^2\left(\tan\left(\frac{x}{2}\right)-1\right)} - \frac{b}{2\,(a+b)^2\left(\tan\left(\frac{x}{2}\right)-1\right)}$$
$$-\frac{4}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3(4a-4b)} + \frac{2}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2(4a-4b)} - \frac{a}{(a-b)^2\left(\tan\left(\frac{x}{2}\right)+1\right)} + \frac{b}{2\,(a-b)^2\left(\tan\left(\frac{x}{2}\right)+1\right)}$$
$$-\frac{2a^3 b\arctan\left(\frac{2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{(a+b)^2\,(a-b)^2\sqrt{-a^2+b^2}}$$

Test results for the 10 problems in "4.6.1.4 (d cot)^n (a+b csc)^m.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{\tan(x)^4}{a + a\csc(x)}\, dx$$

Optimal(type 3, 49 leaves, 5 steps):

$$\frac{x}{a}-\frac{(15-8\csc(x))\tan(x)}{15a}+\frac{(5-4\csc(x))\tan(x)^3}{15a}-\frac{(1-\csc(x))\tan(x)^5}{5a}$$

Result(type 3, 101 leaves):

$$-\frac{1}{6a\left(\tan\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{4a\left(\tan\left(\frac{x}{2}\right)-1\right)^2}+\frac{5}{8a\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}+\frac{2}{5a\left(\tan\left(\frac{x}{2}\right)+1\right)^5}-\frac{1}{a\left(\tan\left(\frac{x}{2}\right)+1\right)^4}$$
$$+\frac{1}{a\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{11}{8a\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int\frac{\cot(x)^6}{a+a\csc(x)}\,dx$$

Optimal(type 3, 43 leaves, 5 steps):

$$-\frac{x}{a}-\frac{3\operatorname{arctanh}(\cos(x))}{8a}+\frac{\cot(x)^3(4-3\csc(x))}{12a}-\frac{\cot(x)(8-3\csc(x))}{8a}$$

Result(type 3, 107 leaves):

$$\frac{\tan\left(\frac{x}{2}\right)^4}{64a}-\frac{\tan\left(\frac{x}{2}\right)^3}{24a}-\frac{\tan\left(\frac{x}{2}\right)^2}{8a}+\frac{5\tan\left(\frac{x}{2}\right)}{8a}-\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}-\frac{1}{64a\tan\left(\frac{x}{2}\right)^4}+\frac{1}{24a\tan\left(\frac{x}{2}\right)^3}+\frac{1}{8a\tan\left(\frac{x}{2}\right)^2}-\frac{5}{8a\tan\left(\frac{x}{2}\right)}$$
$$+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8a}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int\frac{\cot(x)^6}{a+b\csc(x)}\,dx$$

Optimal(type 3, 168 leaves, 16 steps):

$$-\frac{x}{a}-\frac{3\operatorname{arctanh}(\cos(x))}{8b}-\frac{(a^2-3b^2)\operatorname{arctanh}(\cos(x))}{2b^3}-\frac{(a^4-3a^2b^2+3b^4)\operatorname{arctanh}(\cos(x))}{b^5}+\frac{2(a^2-b^2)^{5/2}\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{ab^5}$$
$$+\frac{a\cot(x)}{b^2}+\frac{a(a^2-3b^2)\cot(x)}{b^4}+\frac{a\cot(x)^3}{3b^2}-\frac{3\cot(x)\csc(x)}{8b}-\frac{(a^2-3b^2)\cot(x)\csc(x)}{2b^3}-\frac{\cot(x)\csc(x)^3}{4b}$$

Result(type 3, 362 leaves):

$$\frac{\tan\left(\frac{x}{2}\right)^4}{64\,b}-\frac{\tan\left(\frac{x}{2}\right)^3 a}{24\,b^2}+\frac{\tan\left(\frac{x}{2}\right)^2 a^2}{8\,b^3}-\frac{\tan\left(\frac{x}{2}\right)^2}{4\,b}-\frac{\tan\left(\frac{x}{2}\right) a^3}{2\,b^4}+\frac{9\,a\tan\left(\frac{x}{2}\right)}{8\,b^2}-\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}-\frac{1}{64\tan\left(\frac{x}{2}\right)^4 b}-\frac{a^2}{8\,b^3\tan\left(\frac{x}{2}\right)^2}$$
$$+\frac{1}{4\,b\tan\left(\frac{x}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)a^4}{b^5}-\frac{5\ln\left(\tan\left(\frac{x}{2}\right)\right)a^2}{2\,b^3}+\frac{15\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8\,b}+\frac{a}{24\tan\left(\frac{x}{2}\right)^3 b^2}+\frac{a^3}{2\,b^4\tan\left(\frac{x}{2}\right)}-\frac{9\,a}{8\,b^2\tan\left(\frac{x}{2}\right)}$$
$$-\frac{2\,a^5\arctan\left(\frac{2\,b\tan\left(\frac{x}{2}\right)+2\,a}{2\sqrt{-a^2+b^2}}\right)}{b^5\sqrt{-a^2+b^2}}+\frac{6\,a^3\arctan\left(\frac{2\,b\tan\left(\frac{x}{2}\right)+2\,a}{2\sqrt{-a^2+b^2}}\right)}{b^3\sqrt{-a^2+b^2}}-\frac{6\,a\arctan\left(\frac{2\,b\tan\left(\frac{x}{2}\right)+2\,a}{2\sqrt{-a^2+b^2}}\right)}{b\sqrt{-a^2+b^2}}+\frac{2\,b\arctan\left(\frac{2\,b\tan\left(\frac{x}{2}\right)+2\,a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}$$

Test results for the 25 problems in "4.6.11 (e x)^m (a+b csc(c+d x^n))^p.txt"

Problem 1: Unable to integrate problem.

$$\int x^5\left(a+b\csc\left(d\,x^2+c\right)\right)\mathrm{d}x$$

Optimal(type 4, 115 leaves, 10 steps):

$$\frac{a\,x^6}{6}-\frac{b\,x^4\operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}\right)}{d}+\frac{\mathrm{I}\,b\,x^2\operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}\right)}{d^2}-\frac{\mathrm{I}\,b\,x^2\operatorname{polylog}\left(2,\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}\right)}{d^2}-\frac{b\operatorname{polylog}\left(3,-\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}\right)}{d^3}+\frac{b\operatorname{polylog}\left(3,\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}\right)}{d^3}$$

Result(type 8, 44 leaves):

$$\frac{a\,x^6}{6}+\int\frac{2\,\mathrm{I}\,b\,x^5\,\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}}{\left(\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}\right)^2-1}\,\mathrm{d}x$$

Problem 2: Unable to integrate problem.

$$\int x^3\left(a+b\csc\left(d\,x^2+c\right)\right)\mathrm{d}x$$

Optimal(type 4, 70 leaves, 8 steps):

$$\frac{a\,x^4}{4}-\frac{b\,x^2\operatorname{arctanh}\left(\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}\right)}{d}+\frac{\mathrm{I}\,b\operatorname{polylog}\left(2,-\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}\right)}{2\,d^2}-\frac{\mathrm{I}\,b\operatorname{polylog}\left(2,\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}\right)}{2\,d^2}$$

Result(type 8, 44 leaves):

$$\frac{a\,x^4}{4}+\int\frac{2\,\mathrm{I}\,b\,x^3\,\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}}{\left(\mathrm{e}^{\mathrm{I}\,(d\,x^2+c)}\right)^2-1}\,\mathrm{d}x$$

Problem 4: Unable to integrate problem.

$$\int x^3 \left(a + b\csc\left(d x^2 + c\right)\right)^2 \mathrm{d}x$$

Optimal(type 4, 111 leaves, 10 steps):

$$\frac{a^2 x^4}{4} - \frac{2 a b x^2 \operatorname{arctanh}\left(e^{I\left(d x^2 + c\right)}\right)}{d} - \frac{b^2 x^2 \cot\left(d x^2 + c\right)}{2 d} + \frac{b^2 \ln\left(\sin\left(d x^2 + c\right)\right)}{2 d^2} + \frac{I a b \operatorname{polylog}\left(2, -e^{I\left(d x^2 + c\right)}\right)}{d^2} - \frac{I a b \operatorname{polylog}\left(2, e^{I\left(d x^2 + c\right)}\right)}{d^2}$$

Result(type 8, 85 leaves):

$$\frac{a^2 x^4}{4} - \frac{I x^2 b^2}{d\left(\left(e^{I\left(d x^2 + c\right)}\right)^2 - 1\right)} + \int \frac{2 I b x\left(2 a d x^2 e^{I\left(d x^2 + c\right)} + b\right)}{d\left(\left(e^{I\left(d x^2 + c\right)}\right)^2 - 1\right)} \mathrm{d}x$$

Problem 6: Unable to integrate problem.

$$\int \frac{x^5}{a + b\csc\left(d x^2 + c\right)} \mathrm{d}x$$

Optimal(type 4, 344 leaves, 13 steps):

$$\frac{x^6}{6 a} + \frac{I b x^4 \ln\left(1 - \frac{I a e^{I\left(d x^2 + c\right)}}{b - \sqrt{-a^2 + b^2}}\right)}{2 a d\sqrt{-a^2 + b^2}} - \frac{I b x^4 \ln\left(1 - \frac{I a e^{I\left(d x^2 + c\right)}}{b + \sqrt{-a^2 + b^2}}\right)}{2 a d\sqrt{-a^2 + b^2}} + \frac{b x^2 \operatorname{polylog}\left(2, \frac{I a e^{I\left(d x^2 + c\right)}}{b - \sqrt{-a^2 + b^2}}\right)}{a d^2\sqrt{-a^2 + b^2}} - \frac{b x^2 \operatorname{polylog}\left(2, \frac{I a e^{I\left(d x^2 + c\right)}}{b + \sqrt{-a^2 + b^2}}\right)}{a d^2\sqrt{-a^2 + b^2}}$$
$$+ \frac{I b \operatorname{polylog}\left(3, \frac{I a e^{I\left(d x^2 + c\right)}}{b - \sqrt{-a^2 + b^2}}\right)}{a d^3\sqrt{-a^2 + b^2}} - \frac{I b \operatorname{polylog}\left(3, \frac{I a e^{I\left(d x^2 + c\right)}}{b + \sqrt{-a^2 + b^2}}\right)}{a d^3\sqrt{-a^2 + b^2}}$$

Result(type 8, 68 leaves):

$$\frac{x^6}{6 a} + \int \frac{-2 I b x^5 e^{I\left(d x^2 + c\right)}}{a\left(2 I b e^{I\left(d x^2 + c\right)} + \left(e^{I\left(d x^2 + c\right)}\right)^2 a - a\right)} \mathrm{d}x$$

Problem 8: Unable to integrate problem.

$$\int \frac{x^3}{a + b\csc\left(d x^2 + c\right)} \mathrm{d}x$$

Optimal(type 4, 231 leaves, 11 steps):

$$\frac{x^4}{4 a} + \frac{I b x^2 \ln\left(1 - \frac{I a e^{I\left(d x^2 + c\right)}}{b - \sqrt{-a^2 + b^2}}\right)}{2 a d\sqrt{-a^2 + b^2}} - \frac{I b x^2 \ln\left(1 - \frac{I a e^{I\left(d x^2 + c\right)}}{b + \sqrt{-a^2 + b^2}}\right)}{2 a d\sqrt{-a^2 + b^2}} + \frac{b \operatorname{polylog}\left(2, \frac{I a e^{I\left(d x^2 + c\right)}}{b - \sqrt{-a^2 + b^2}}\right)}{2 a d^2\sqrt{-a^2 + b^2}} - \frac{b \operatorname{polylog}\left(2, \frac{I a e^{I\left(d x^2 + c\right)}}{b + \sqrt{-a^2 + b^2}}\right)}{2 a d^2\sqrt{-a^2 + b^2}}$$

Result(type 8, 68 leaves):

$$\frac{x^4}{4a} + \int \frac{-2\,I\,b\,x^3\,e^{I(d x^2+c)}}{a\left(2\,I\,b\,e^{I(d x^2+c)} + \left(e^{I(d x^2+c)}\right)^2 a - a\right)}\,dx$$

Problem 10: Unable to integrate problem.

$$\int \frac{x^5}{\left(a + b\csc(d x^2 + c)\right)^2}\,dx$$

Optimal(type 4, 999 leaves, 31 steps):

$$-\frac{I\,b^3\,\mathrm{polylog}\left(3, \frac{I\,a\,e^{I(d x^2+c)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,(-a^2+b^2)^{3/2}\,d^3} + \frac{x^6}{6\,a^2} + \frac{b^2\,x^2\ln\left(1+\frac{a\,e^{I(d x^2+c)}}{I\,b-\sqrt{a^2-b^2}}\right)}{a^2\,(a^2-b^2)\,d^2} + \frac{b^2\,x^2\ln\left(1+\frac{a\,e^{I(d x^2+c)}}{I\,b+\sqrt{a^2-b^2}}\right)}{a^2\,(a^2-b^2)\,d^2} + \frac{I\,b\,x^4\ln\left(1-\frac{I\,a\,e^{I(d x^2+c)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,d\sqrt{-a^2+b^2}}$$

$$-\frac{I\,b\,x^4\ln\left(1-\frac{I\,a\,e^{I(d x^2+c)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,d\sqrt{-a^2+b^2}} - \frac{I\,b^2\,\mathrm{polylog}\left(2, -\frac{a\,e^{I(d x^2+c)}}{I\,b+\sqrt{a^2-b^2}}\right)}{a^2\,(a^2-b^2)\,d^3} + \frac{I\,b^3\,x^4\ln\left(1-\frac{I\,a\,e^{I(d x^2+c)}}{b+\sqrt{-a^2+b^2}}\right)}{2\,a^2\,(-a^2+b^2)^{3/2}\,d} - \frac{b^3\,x^2\,\mathrm{polylog}\left(2, \frac{I\,a\,e^{I(d x^2+c)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,(-a^2+b^2)^{3/2}\,d^2}$$

$$+\frac{b^3\,x^2\,\mathrm{polylog}\left(2, \frac{I\,a\,e^{I(d x^2+c)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,(-a^2+b^2)^{3/2}\,d^2} - \frac{2\,I\,b\,\mathrm{polylog}\left(3, \frac{I\,a\,e^{I(d x^2+c)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,d^3\sqrt{-a^2+b^2}} + \frac{I\,b^3\,\mathrm{polylog}\left(3, \frac{I\,a\,e^{I(d x^2+c)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,(-a^2+b^2)^{3/2}\,d^3}$$

$$-\frac{b^2\,x^4\cos(d x^2+c)}{2\,a\,(a^2-b^2)\,d\,(b+a\sin(d x^2+c))} - \frac{I\,b^2\,x^4}{2\,a^2\,(a^2-b^2)\,d} - \frac{I\,b^3\,x^4\ln\left(1-\frac{I\,a\,e^{I(d x^2+c)}}{b-\sqrt{-a^2+b^2}}\right)}{2\,a^2\,(-a^2+b^2)^{3/2}\,d} + \frac{2\,b\,x^2\,\mathrm{polylog}\left(2, \frac{I\,a\,e^{I(d x^2+c)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,d^2\sqrt{-a^2+b^2}}$$

$$-\frac{2\,b\,x^2\,\mathrm{polylog}\left(2, \frac{I\,a\,e^{I(d x^2+c)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\,d^2\sqrt{-a^2+b^2}} - \frac{I\,b^2\,\mathrm{polylog}\left(2, -\frac{a\,e^{I(d x^2+c)}}{I\,b-\sqrt{a^2-b^2}}\right)}{a^2\,(a^2-b^2)\,d^3} + \frac{2\,I\,b\,\mathrm{polylog}\left(3, \frac{I\,a\,e^{I(d x^2+c)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,d^3\sqrt{-a^2+b^2}}$$

Result(type 8, 215 leaves):

$$\frac{x^6}{6\,a^2} - \frac{I\,b^2\,x^4\left(I\,a + b\,e^{I(d x^2+c)}\right)}{a^2\,(-a^2+b^2)\,d\left(2\,b\,e^{I(d x^2+c)} - I\left(e^{I(d x^2+c)}\right)^2 a + I\,a\right)} + \int \frac{-2\,I\,b\,x^3\left(2\,x^2\,a^2\,d\,e^{I(d x^2+c)} - x^2\,b^2\,d\,e^{I(d x^2+c)} + 2\,I\,b^2\,e^{I(d x^2+c)} - 2\,b\,a\right)}{a^2\,(a^2-b^2)\,d\left(2\,I\,b\,e^{I(d x^2+c)} + \left(e^{I(d x^2+c)}\right)^2 a - a\right)}\,dx$$

Problem 14: Unable to integrate problem.

$$\int \frac{x^3}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx$$

Optimal(type 4, 905 leaves, 23 steps):

$$\frac{x^4}{4a} - \frac{10080\,I\,b\,\mathrm{polylog}\left(7, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a\,d^7\sqrt{-a^2+b^2}} + \frac{1680\,I\,b\,x^{3/2}\,\mathrm{polylog}\left(5, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\,d^5\sqrt{-a^2+b^2}} + \frac{14\,b\,x^3\,\mathrm{polylog}\left(2, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\,d^2\sqrt{-a^2+b^2}}$$

$$- \frac{14\,b\,x^3\,\mathrm{polylog}\left(2, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\,d^2\sqrt{-a^2+b^2}} + \frac{10080\,I\,b\,\mathrm{polylog}\left(7, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)\sqrt{x}}{a\,d^7\sqrt{-a^2+b^2}} + \frac{2\,I\,b\,x^{7/2}\ln\left(1 - \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\,d\sqrt{-a^2+b^2}}$$

$$- \frac{420\,b\,x^2\,\mathrm{polylog}\left(4, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\,d^4\sqrt{-a^2+b^2}} + \frac{420\,b\,x^2\,\mathrm{polylog}\left(4, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\,d^4\sqrt{-a^2+b^2}} - \frac{1680\,I\,b\,x^{3/2}\,\mathrm{polylog}\left(5, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\,d^5\sqrt{-a^2+b^2}}$$

$$- \frac{84\,I\,b\,x^{5/2}\,\mathrm{polylog}\left(3, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\,d^3\sqrt{-a^2+b^2}} + \frac{5040\,b\,x\,\mathrm{polylog}\left(6, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\,d^6\sqrt{-a^2+b^2}} - \frac{5040\,b\,x\,\mathrm{polylog}\left(6, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\,d^6\sqrt{-a^2+b^2}}$$

$$- \frac{10080\,b\,\mathrm{polylog}\left(8, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\,d^8\sqrt{-a^2+b^2}} + \frac{10080\,b\,\mathrm{polylog}\left(8, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\,d^8\sqrt{-a^2+b^2}} - \frac{2\,I\,b\,x^{7/2}\ln\left(1 - \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\,d\sqrt{-a^2+b^2}}$$

$$+ \frac{84\,I\,b\,x^{5/2}\,\mathrm{polylog}\left(3, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\,d^3\sqrt{-a^2+b^2}}$$

Result(type 8, 20 leaves):

$$\int \frac{x^3}{a+b\csc(c+d\sqrt{x})}\,dx$$

Problem 16: Unable to integrate problem.

$$\int \frac{x^2}{\left(a+b\csc(c+d\sqrt{x})\right)^2}\,dx$$

Optimal(type 4, 2040 leaves, 49 steps):

$$-\frac{240\,b^2\,\mathrm{polylog}\left(5, -\frac{a\,e^{I(c+d\sqrt{x})}}{I\,b-\sqrt{a^2-b^2}}\right)}{a^2\,(a^2-b^2)\,d^6} - \frac{240\,b^2\,\mathrm{polylog}\left(5, -\frac{a\,e^{I(c+d\sqrt{x})}}{I\,b+\sqrt{a^2-b^2}}\right)}{a^2\,(a^2-b^2)\,d^6} - \frac{240\,b^3\,\mathrm{polylog}\left(6, \frac{I\,a\,e^{I(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\,(-a^2+b^2)^{3/2}\,d^6}$$

$$+\frac{240\,b^{3}\,\mathrm{polylog}\left(6,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b+\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,(-a^{2}+b^{2})^{3/2}\,d^{6}}+\frac{480\,b\,\mathrm{polylog}\left(6,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b-\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,d^{6}\sqrt{-a^{2}+b^{2}}}-\frac{480\,b\,\mathrm{polylog}\left(6,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b+\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,d^{6}\sqrt{-a^{2}+b^{2}}}+\frac{x^{3}}{3\,a^{2}}$$

$$+\frac{10\,b^{2}\,x^{2}\ln\left(1+\frac{a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{\mathrm{I}\,b-\sqrt{a^{2}-b^{2}}}\right)}{a^{2}\,(a^{2}-b^{2})\,d^{2}}+\frac{10\,b^{2}\,x^{2}\ln\left(1+\frac{a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{\mathrm{I}\,b+\sqrt{a^{2}-b^{2}}}\right)}{a^{2}\,(a^{2}-b^{2})\,d^{2}}-\frac{10\,b^{3}\,x^{2}\,\mathrm{polylog}\left(2,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b-\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,(-a^{2}+b^{2})^{3/2}\,d^{2}}$$

$$+\frac{10\,b^{3}\,x^{2}\,\mathrm{polylog}\left(2,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b+\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,(-a^{2}+b^{2})^{3/2}\,d^{2}}+\frac{120\,b^{2}\,x\,\mathrm{polylog}\left(3,-\frac{a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{\mathrm{I}\,b-\sqrt{a^{2}-b^{2}}}\right)}{a^{2}\,(a^{2}-b^{2})\,d^{4}}+\frac{120\,b^{2}\,x\,\mathrm{polylog}\left(3,-\frac{a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{\mathrm{I}\,b+\sqrt{a^{2}-b^{2}}}\right)}{a^{2}\,(a^{2}-b^{2})\,d^{4}}$$

$$+\frac{120\,b^{3}\,x\,\mathrm{polylog}\left(4,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b-\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,(-a^{2}+b^{2})^{3/2}\,d^{4}}-\frac{120\,b^{3}\,x\,\mathrm{polylog}\left(4,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b+\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,(-a^{2}+b^{2})^{3/2}\,d^{4}}+\frac{20\,b\,x^{2}\,\mathrm{polylog}\left(2,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b-\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,d^{2}\sqrt{-a^{2}+b^{2}}}$$

$$-\frac{20\,b\,x^{2}\,\mathrm{polylog}\left(2,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b+\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,d^{2}\sqrt{-a^{2}+b^{2}}}-\frac{240\,b\,x\,\mathrm{polylog}\left(4,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b-\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,d^{4}\sqrt{-a^{2}+b^{2}}}+\frac{240\,b\,x\,\mathrm{polylog}\left(4,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b+\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,d^{4}\sqrt{-a^{2}+b^{2}}}-\frac{2\,\mathrm{I}\,b^{2}\,x^{5/2}}{a^{2}\,(a^{2}-b^{2})\,d}$$

$$-\frac{240\,\mathrm{I}\,b^{3}\,\mathrm{polylog}\left(5,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b+\sqrt{-a^{2}+b^{2}}}\right)\sqrt{x}}{a^{2}\,(-a^{2}+b^{2})^{3/2}\,d^{5}}+\frac{480\,\mathrm{I}\,b\,\mathrm{polylog}\left(5,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b+\sqrt{-a^{2}+b^{2}}}\right)\sqrt{x}}{a^{2}\,d^{5}\sqrt{-a^{2}+b^{2}}}+\frac{4\,\mathrm{I}\,b\,x^{5/2}\ln\left(1-\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b-\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,d\sqrt{-a^{2}+b^{2}}}$$

$$+\frac{240\,\mathrm{I}\,b^{3}\,\mathrm{polylog}\left(5,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b-\sqrt{-a^{2}+b^{2}}}\right)\sqrt{x}}{a^{2}\,(-a^{2}+b^{2})^{3/2}\,d^{5}}+\frac{80\,\mathrm{I}\,b\,x^{3/2}\,\mathrm{polylog}\left(3,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b-\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,d^{3}\sqrt{-a^{2}+b^{2}}}+\frac{2\,\mathrm{I}\,b^{3}\,x^{5/2}\ln\left(1-\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b+\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,(-a^{2}+b^{2})^{3/2}\,d}$$

$$+\frac{240\,\mathrm{I}\,b^{2}\,\mathrm{polylog}\left(4,-\frac{a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{\mathrm{I}\,b-\sqrt{a^{2}-b^{2}}}\right)\sqrt{x}}{a^{2}\,(a^{2}-b^{2})\,d^{5}}+\frac{40\,\mathrm{I}\,b^{3}\,x^{3/2}\,\mathrm{polylog}\left(3,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b+\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,(-a^{2}+b^{2})^{3/2}\,d^{3}}+\frac{240\,\mathrm{I}\,b^{2}\,\mathrm{polylog}\left(4,-\frac{a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{\mathrm{I}\,b+\sqrt{a^{2}-b^{2}}}\right)\sqrt{x}}{a^{2}\,(a^{2}-b^{2})\,d^{5}}$$

$$-\frac{480\,\mathrm{I}\,b\,\mathrm{polylog}\left(5,\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b-\sqrt{-a^{2}+b^{2}}}\right)\sqrt{x}}{a^{2}\,d^{5}\sqrt{-a^{2}+b^{2}}}-\frac{2\,b^{2}\,x^{5/2}\cos\left(c+d\sqrt{x}\right)}{a\,(a^{2}-b^{2})\,d\,\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}-\frac{2\,\mathrm{I}\,b^{3}\,x^{5/2}\ln\left(1-\frac{\mathrm{I}\,a\,\mathrm{e}^{\mathrm{I}\,(c+d\sqrt{x})}}{b-\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}\,(-a^{2}+b^{2})^{3/2}\,d}$$

$$-\frac{40 I b^2 x^{3/2} \operatorname{polylog}\left(2, -\frac{a e^{I(c+d\sqrt{x})}}{I b - \sqrt{a^2-b^2}}\right)}{a^2 (a^2-b^2) d^3} - \frac{40 I b^2 x^{3/2} \operatorname{polylog}\left(2, -\frac{a e^{I(c+d\sqrt{x})}}{I b + \sqrt{a^2-b^2}}\right)}{a^2 (a^2-b^2) d^3} - \frac{40 I b^3 x^{3/2} \operatorname{polylog}\left(3, \frac{I a e^{I(c+d\sqrt{x})}}{b - \sqrt{-a^2+b^2}}\right)}{a^2 (-a^2+b^2)^{3/2} d^3}$$

$$-\frac{4 I b x^{5/2} \ln\left(1 - \frac{I a e^{I(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 d \sqrt{-a^2+b^2}} - \frac{80 I b x^{3/2} \operatorname{polylog}\left(3, \frac{I a e^{I(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 d^3 \sqrt{-a^2+b^2}}$$

Result(type 8, 20 leaves):

$$\int \frac{x^2}{\left(a + b \csc\left(c + d\sqrt{x}\right)\right)^2} \, dx$$

Problem 18: Unable to integrate problem.

$$\int x^{3/2} \left(a + b \csc\left(c + d\sqrt{x}\right)\right)^2 dx$$

Optimal(type 4, 344 leaves, 21 steps):

$$\frac{6 I b^2 \operatorname{polylog}\left(4, e^{2 I(c+d\sqrt{x})}\right)}{d^5} + \frac{2 a^2 x^{5/2}}{5} - \frac{8 a b x^2 \operatorname{arctanh}\left(e^{I(c+d\sqrt{x})}\right)}{d} - \frac{2 b^2 x^2 \cot\left(c+d\sqrt{x}\right)}{d} + \frac{8 b^2 x^{3/2} \ln\left(1 - e^{2 I(c+d\sqrt{x})}\right)}{d^2} - \frac{2 I b^2 x^2}{d}$$

$$+ \frac{16 I a b x^{3/2} \operatorname{polylog}\left(2, -e^{I(c+d\sqrt{x})}\right)}{d^2} - \frac{16 I a b x^{3/2} \operatorname{polylog}\left(2, e^{I(c+d\sqrt{x})}\right)}{d^2} - \frac{48 a b x \operatorname{polylog}\left(3, -e^{I(c+d\sqrt{x})}\right)}{d^3} + \frac{48 a b x \operatorname{polylog}\left(3, e^{I(c+d\sqrt{x})}\right)}{d^3}$$

$$- \frac{12 I b^2 x \operatorname{polylog}\left(2, e^{2 I(c+d\sqrt{x})}\right)}{d^3} + \frac{96 a b \operatorname{polylog}\left(5, -e^{I(c+d\sqrt{x})}\right)}{d^5} - \frac{96 a b \operatorname{polylog}\left(5, e^{I(c+d\sqrt{x})}\right)}{d^5} + \frac{12 b^2 \operatorname{polylog}\left(3, e^{2 I(c+d\sqrt{x})}\right) \sqrt{x}}{d^4}$$

$$+ \frac{96 I a b \operatorname{polylog}\left(4, e^{I(c+d\sqrt{x})}\right) \sqrt{x}}{d^4} - \frac{96 I a b \operatorname{polylog}\left(4, -e^{I(c+d\sqrt{x})}\right) \sqrt{x}}{d^4}$$

Result(type 8, 20 leaves):

$$\int x^{3/2} \left(a + b \csc\left(c + d\sqrt{x}\right)\right)^2 dx$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{(e x)^{-1+2n}}{a + b \csc(c + d x^n)} \, dx$$

Optimal(type 4, 310 leaves, 12 steps):

$$\frac{(ex)^{2n}}{2aen}+\frac{\mathrm{I}b(ex)^{2n}\ln\left(1-\frac{\mathrm{I}a\,\mathrm{e}^{\mathrm{I}(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{adenx^n\sqrt{-a^2+b^2}}-\frac{\mathrm{I}b(ex)^{2n}\ln\left(1-\frac{\mathrm{I}a\,\mathrm{e}^{\mathrm{I}(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{adenx^n\sqrt{-a^2+b^2}}+\frac{b(ex)^{2n}\operatorname{polylog}\left(2,\frac{\mathrm{I}a\,\mathrm{e}^{\mathrm{I}(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{ad^2enx^{2n}\sqrt{-a^2+b^2}}-\frac{b(ex)^{2n}\operatorname{polylog}\left(2,\frac{\mathrm{I}a\,\mathrm{e}^{\mathrm{I}(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{ad^2enx^{2n}\sqrt{-a^2+b^2}}$$

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Result(type 4, 1335 leaves):
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$$\frac{x\,\mathrm{e}^{-\frac{(-1+2n)\left(\mathrm{I}\pi\operatorname{csgn}(\mathrm{I}ex)^3-\mathrm{I}\pi\operatorname{csgn}(\mathrm{I}ex)^2\operatorname{csgn}(\mathrm{I}e)-\mathrm{I}\pi\operatorname{csgn}(\mathrm{I}ex)^2\operatorname{csgn}(\mathrm{I}x)+\mathrm{I}\pi\operatorname{csgn}(\mathrm{I}ex)\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}x)-2\ln(x)-2\ln(e)\right)}{2}}}{2an}$$

$$-\frac{1}{aend\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}\left(b\left(e^n\right)^2\sqrt{(-1)^{\operatorname{csgn}(\mathrm{I}ex)^3}}\,(-1)^{\frac{\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)}{2}}x^n\ln\left(\frac{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b+a\,\mathrm{e}^{\mathrm{I}(dx^n+2c)}-\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b-\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}\right)\right.$$
$$\left.\mathrm{e}^{-\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)}\,\mathrm{e}^{\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{-\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}ex)^3}\,\mathrm{e}^{-\frac{\mathrm{I}}{2}\pi\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{-\frac{\mathrm{I}}{2}\pi\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{\mathrm{I}c}\right)$$

$$+\frac{1}{aend\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}\left(b\left(e^n\right)^2\sqrt{(-1)^{\operatorname{csgn}(\mathrm{I}ex)^3}}\,(-1)^{\frac{\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)}{2}}x^n\ln\left(\frac{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b+a\,\mathrm{e}^{\mathrm{I}(dx^n+2c)}+\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b+\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}\right)\right.$$
$$\left.\mathrm{e}^{-\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)}\,\mathrm{e}^{\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{-\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}ex)^3}\,\mathrm{e}^{-\frac{\mathrm{I}}{2}\pi\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{-\frac{\mathrm{I}}{2}\pi\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{\mathrm{I}c}\right)$$

$$+\frac{1}{aend^2\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}\left(\mathrm{I}b\left(e^n\right)^2\sqrt{(-1)^{\operatorname{csgn}(\mathrm{I}ex)^3}}\,(-1)^{\frac{\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)}{2}}\operatorname{dilog}\left(\frac{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b}{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b-\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}\right.\right.$$
$$\left.+\frac{a\,\mathrm{e}^{\mathrm{I}(dx^n+2c)}}{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b-\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}-\frac{\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b-\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}\right)$$
$$\left.\mathrm{e}^{-\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)}\,\mathrm{e}^{\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{-\mathrm{I}\pi n\operatorname{csgn}(\mathrm{I}ex)^3}\,\mathrm{e}^{-\frac{\mathrm{I}}{2}\pi\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{-\frac{\mathrm{I}}{2}\pi\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)^2}\,\mathrm{e}^{\mathrm{I}c}\right)$$

$$-\frac{1}{aend^2\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}\left(\mathrm{I}b\left(e^n\right)^2\sqrt{(-1)^{\operatorname{csgn}(\mathrm{I}ex)^3}}\,(-1)^{\frac{\operatorname{csgn}(\mathrm{I}e)\operatorname{csgn}(\mathrm{I}x)\operatorname{csgn}(\mathrm{I}ex)}{2}}\operatorname{dilog}\left(\frac{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b}{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b+\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}\right.\right.$$
$$\left.+\frac{a\,\mathrm{e}^{\mathrm{I}(dx^n+2c)}}{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b+\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}+\frac{\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}{\mathrm{I}\mathrm{e}^{\mathrm{I}c}b+\sqrt{-\mathrm{e}^{2\mathrm{I}c}b^2+\mathrm{e}^{2\mathrm{I}c}a^2}}\right)$$

$$e^{-I\pi n \operatorname{csgn}(I e) \operatorname{csgn}(I x) \operatorname{csgn}(I e x)} e^{I\pi n \operatorname{csgn}(I e) \operatorname{csgn}(I e x)^2} e^{I\pi n \operatorname{csgn}(I x) \operatorname{csgn}(I e x)^2} e^{-I\pi n \operatorname{csgn}(I e x)^3} e^{-\frac{I}{2}\pi \operatorname{csgn}(I e) \operatorname{csgn}(I e x)^2} e^{-\frac{I}{2}\pi \operatorname{csgn}(I x) \operatorname{csgn}(I e x)^2} e^{I c}\Big)$$

Problem 25: Unable to integrate problem.

$$\int \frac{(e x)^{-1+3n}}{a + b\csc(c + d x^n)}\, dx$$

Optimal(type 4, 459 leaves, 14 steps):

$$\frac{(ex)^{3n}}{3aen} + \frac{Ib(ex)^{3n}\ln\left(1 - \frac{Iae^{I(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{aden x^n\sqrt{-a^2+b^2}} - \frac{Ib(ex)^{3n}\ln\left(1 - \frac{Iae^{I(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{aden x^n\sqrt{-a^2+b^2}} + \frac{2b(ex)^{3n}\operatorname{polylog}\left(2, \frac{Iae^{I(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{ad^2en x^{2n}\sqrt{-a^2+b^2}}$$
$$- \frac{2b(ex)^{3n}\operatorname{polylog}\left(2, \frac{Iae^{I(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{ad^2en x^{2n}\sqrt{-a^2+b^2}} + \frac{2Ib(ex)^{3n}\operatorname{polylog}\left(3, \frac{Iae^{I(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{ad^3en x^{3n}\sqrt{-a^2+b^2}} - \frac{2Ib(ex)^{3n}\operatorname{polylog}\left(3, \frac{Iae^{I(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{ad^3en x^{3n}\sqrt{-a^2+b^2}}$$

Result(type 8, 172 leaves):

$$\frac{x\, e^{(-1+3n)\left(\ln(e)+\ln(x) - \frac{I\pi \operatorname{csgn}(I e x)(-\operatorname{csgn}(I e x)+\operatorname{csgn}(I e))(-\operatorname{csgn}(I e x)+\operatorname{csgn}(I x))}{2}\right)}}{3an} +$$
$$\int \frac{-2Ib\, e^{(-1+3n)\left(\ln(e)+\ln(x) - \frac{I\pi \operatorname{csgn}(I e x)(-\operatorname{csgn}(I e x)+\operatorname{csgn}(I e))(-\operatorname{csgn}(I e x)+\operatorname{csgn}(I x))}{2}\right)} e^{I(c+d e^{\ln(x) n})}}{a\left(2Ib\, e^{I(c+d e^{\ln(x) n})} + a\left(e^{I(c+d e^{\ln(x) n})}\right)^2 - a\right)}\, dx$$

Test results for the 10 problems in "4.6.3.1 (a+b csc)^m (d csc)^n (A+B csc).txt"

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + a\csc(dx + c))(A - A\csc(dx + c))}{\csc(dx + c)^3}\, dx$$

Optimal(type 3, 15 leaves, 3 steps):

$$\frac{aA\cos(dx + c)^3}{3d}$$

Result(type 3, 34 leaves):

$$\frac{-\frac{Aa(2 + \sin(dx + c)^2)\cos(dx + c)}{3} + \cos(dx + c)Aa}{d}$$

Test results for the 1 problems in "4.6.4.2 (a+b csc)^m (d csc)^n (A+B csc+C csc^2).txt"

Test results for the 10 problems in "4.6.7 (d trig)^m (a+b (c csc)^n)^p.txt"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b\csc(dx+c)^2\right)^3}\, dx$$

Optimal(type 3, 130 leaves, 6 steps):

$$\frac{x}{a^3}+\frac{b\cot(dx+c)}{4a(a+b)d\left(a+b+b\cot(dx+c)^2\right)^2}+\frac{b(7a+4b)\cot(dx+c)}{8a^2(a+b)^2d\left(a+b+b\cot(dx+c)^2\right)}+\frac{\left(15a^2+20ba+8b^2\right)\arctan\left(\frac{\cot(dx+c)\sqrt{b}}{\sqrt{a+b}}\right)\sqrt{b}}{8a^3(a+b)^{5/2}d}$$

Result(type 3, 362 leaves):

$$\frac{\arctan(\tan(dx+c))}{da^3}+\frac{9b\tan(dx+c)^3}{8da\left(a\tan(dx+c)^2+\tan(dx+c)^2b+b\right)^2(a+b)}+\frac{b^2\tan(dx+c)^3}{2da^2\left(a\tan(dx+c)^2+\tan(dx+c)^2b+b\right)^2(a+b)}+\frac{7b^2\tan(dx+c)}{8da\left(a\tan(dx+c)^2+\tan(dx+c)^2b+b\right)^2\left(a^2+2ba+b^2\right)}+\frac{b^3\tan(dx+c)}{2da^2\left(a\tan(dx+c)^2+\tan(dx+c)^2b+b\right)^2\left(a^2+2ba+b^2\right)}-\frac{15b\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{(a+b)b}}\right)}{8da\left(a^2+2ba+b^2\right)\sqrt{(a+b)b}}-\frac{5b^2\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{(a+b)b}}\right)}{2da^2\left(a^2+2ba+b^2\right)\sqrt{(a+b)b}}-\frac{b^3\arctan\left(\frac{\tan(dx+c)(a+b)}{\sqrt{(a+b)b}}\right)}{da^3\left(a^2+2ba+b^2\right)\sqrt{(a+b)b}}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \left(a+b\csc(dx+c)^2\right)^{3/2}\, dx$$

Optimal(type 3, 101 leaves, 7 steps):

$$-\frac{a^{3/2}\arctan\left(\frac{\cot(dx+c)\sqrt{a}}{\sqrt{a+b+b\cot(dx+c)^2}}\right)}{d}-\frac{(3a+b)\operatorname{arctanh}\left(\frac{\cot(dx+c)\sqrt{b}}{\sqrt{a+b+b\cot(dx+c)^2}}\right)\sqrt{b}}{2d}-\frac{b\cot(dx+c)\sqrt{a+b+b\cot(dx+c)^2}}{2d}$$

Result(type 3, 1285 leaves):

$$-\frac{1}{4d\sqrt{-a}\sin(dx+c)^3\left(-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}\right)^{3/2}}\Bigg(\left(\frac{\cos(dx+c)^2a-a-b}{\cos(dx+c)^2-1}\right)^{3/2}(-1+\cos(dx+c))^2\Bigg(b^{3/2}\cos(dx+c)\sqrt{-a}\ln\Bigg(-\frac{2(-1+\cos(dx+c))\left(\sqrt{b}\cos(dx+c)\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}+\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}\sqrt{b}+a\cos(dx+c)+a+b\right)}{\sqrt{b}\sin(dx+c)^2}\Bigg)$$

$$-b^{3/2}\cos(dx+c)\sqrt{-a}\,\ln\left(-\frac{4\left(\sqrt{b}\cos(dx+c)\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}+\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}\sqrt{b}-a\cos(dx+c)+a+b\right)}{-1+\cos(dx+c)}\right)$$

$$-b^{3/2}\ln\Bigg($$

$$-\frac{2(-1+\cos(dx+c))\left(\sqrt{b}\cos(dx+c)\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}+\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}\sqrt{b}+a\cos(dx+c)+a+b\right)}{\sqrt{b}\sin(dx+c)^2}\Bigg)\sqrt{-a}$$

$$+b^{3/2}\ln\left(-\frac{4\left(\sqrt{b}\cos(dx+c)\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}+\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}\sqrt{b}-a\cos(dx+c)+a+b\right)}{-1+\cos(dx+c)}\right)\sqrt{-a}+3\sqrt{b}\cos(dx$$

$$+c)\sqrt{-a}\,\ln\Bigg($$

$$-\frac{2(-1+\cos(dx+c))\left(\sqrt{b}\cos(dx+c)\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}+\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}\sqrt{b}+a\cos(dx+c)+a+b\right)}{\sqrt{b}\sin(dx+c)^2}\Bigg)a$$

$$-3\sqrt{b}\cos(dx+c)\sqrt{-a}\,\ln\left(-\frac{4\left(\sqrt{b}\cos(dx+c)\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}+\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}\sqrt{b}-a\cos(dx+c)+a+b\right)}{-1+\cos(dx+c)}\right)a$$

$$-3\sqrt{b}\,\ln\Bigg($$

$$-\frac{2(-1+\cos(dx+c))\left(\sqrt{b}\cos(dx+c)\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}+\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}\sqrt{b}+a\cos(dx+c)+a+b\right)}{\sqrt{b}\sin(dx+c)^2}\Bigg)a\sqrt{-a}$$

$$+3\sqrt{b}\,\ln\left(-\frac{4\left(\sqrt{b}\cos(dx+c)\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}+\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}\sqrt{b}-a\cos(dx+c)+a+b\right)}{-1+\cos(dx+c)}\right)a\sqrt{-a}+2\cos(dx$$

$$+c)\sqrt{-a}\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}\,b-4\cos(dx+c)\ln\Bigg(4\cos(dx+c)\sqrt{-a}\sqrt{-\frac{\cos(dx+c)^2a-a-b}{(\cos(dx+c)+1)^2}}-4a\cos(dx+c)$$

$$+4\sqrt{-a}\,\sqrt{-\frac{\cos(dx+c)^2 a-a-b}{(\cos(dx+c)+1)^2}}\Bigg)a^2+4a^2\ln\Bigg(4\cos(dx+c)\sqrt{-a}\,\sqrt{-\frac{\cos(dx+c)^2 a-a-b}{(\cos(dx+c)+1)^2}}-4a\cos(dx+c)$$
$$+4\sqrt{-a}\,\sqrt{-\frac{\cos(dx+c)^2 a-a-b}{(\cos(dx+c)+1)^2}}\Bigg)\Bigg)\Bigg)$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b\csc(dx+c)^2)^{7/2}}\,dx$$

Optimal(type 3, 162 leaves, 7 steps):

$$-\frac{\arctan\left(\frac{\cot(dx+c)\sqrt{a}}{\sqrt{a+b+b\cot(dx+c)^2}}\right)}{a^{7/2}d}+\frac{b\cot(dx+c)}{5a(a+b)d\left(a+b+b\cot(dx+c)^2\right)^{5/2}}+\frac{b(9a+5b)\cot(dx+c)}{15a^2(a+b)^2 d\left(a+b+b\cot(dx+c)^2\right)^{3/2}}$$
$$+\frac{b\left(33a^2+40ba+15b^2\right)\cot(dx+c)}{15a^3(a+b)^3 d\sqrt{a+b+b\cot(dx+c)^2}}$$

Result(type ?, 4814 leaves): Display of huge result suppressed!

Problem 6: Result more than twice size of optimal antiderivative.

$$\int (1+\csc(x)^2)^{3/2}\,dx$$

Optimal(type 3, 37 leaves, 6 steps):

$$-2\,\mathrm{arcsinh}\left(\frac{\cot(x)\sqrt{2}}{2}\right)-\arctan\left(\frac{\cot(x)}{\sqrt{2+\cot(x)^2}}\right)-\frac{\cot(x)\sqrt{2+\cot(x)^2}}{2}$$

Result(type 3, 311 leaves):

$$-\frac{1}{2\sin(x)^3\left(-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}\right)^{3/2}}\Bigg(\left(\frac{\cos(x)^2-2}{\cos(x)^2-1}\right)^{3/2}(-1+\cos(x))^2\Bigg(\cos(x)\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}+2\cos(x)\ln\Bigg($$
$$-\frac{2\left(\cos(x)^2\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}+\cos(x)^2+\cos(x)-\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}-2\right)}{\sin(x)^2}\Bigg)-2\cos(x)\,\mathrm{arctanh}\left(\frac{\cos(x)^2-3\cos(x)+2}{\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}\,\sin(x)^2}\right)$$

$$+2\cos(x)\arctan\left(\frac{\cos(x)(-1+\cos(x))}{\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}\sin(x)^2}\right)-2\ln\left(-\frac{2\left(\cos(x)^2\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}+\cos(x)^2+\cos(x)-\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}-2\right)}{\sin(x)^2}\right)$$

$$+2\,\text{arctanh}\left(\frac{\cos(x)^2-3\cos(x)+2}{\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}\sin(x)^2}\right)-2\arctan\left(\frac{\cos(x)(-1+\cos(x))}{\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}\sin(x)^2}\right)\Bigg)\Bigg)\Bigg)$$

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Problem 7: Result more than twice size of optimal antiderivative.
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$$\int\sqrt{1+\csc(x)^2}\,dx$$

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Optimal(type 3, 25 leaves, 5 steps):
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$$-\text{arcsinh}\left(\frac{\cot(x)\sqrt{2}}{2}\right)-\arctan\left(\frac{\cot(x)}{\sqrt{2+\cot(x)^2}}\right)$$

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Result(type 3, 165 leaves):
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$$-\frac{1}{4\sin(x)\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}}\left(\sqrt{4}\sqrt{\frac{\cos(x)^2-2}{\cos(x)^2-1}}(-1+\cos(x))\left(\ln\left(\right.\right.\right.$$

$$-\frac{2\left(\cos(x)^2\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}+\cos(x)^2+\cos(x)-\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}-2\right)}{\sin(x)^2}\Bigg)-\text{arctanh}\left(\frac{\cos(x)^2-3\cos(x)+2}{\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}\sin(x)^2}\right)$$

$$+2\arctan\left(\frac{\cos(x)(-1+\cos(x))}{\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}\sin(x)^2}\right)\Bigg)\Bigg)$$

```
Problem 8: Result more than twice size of optimal antiderivative.
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$$\int\frac{1}{\sqrt{1+\csc(x)^2}}\,dx$$

Optimal(type 3, 14 leaves, 3 steps):

$$-\arctan\left(\frac{\cot(x)}{\sqrt{2+\cot(x)^2}}\right)$$

Result(type 3, 71 leaves):

$$-\frac{\sin(x)\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}\arctan\left(\frac{\cos(x)(-1+\cos(x))}{\sqrt{-\frac{\cos(x)^2-2}{(\cos(x)+1)^2}}\sin(x)^2}\right)}{\sqrt{\frac{\cos(x)^2-2}{\cos(x)^2-1}}(-1+\cos(x))}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int (1-\csc(x)^2)^{3/2}\,dx$$

Optimal(type 3, 27 leaves, 4 steps):

$$\frac{\cot(x)\sqrt{-\cot(x)^2}}{2}+\ln(\sin(x))\sqrt{-\cot(x)^2}\tan(x)$$

Result(type 3, 88 leaves):

$$-\frac{1}{8\cos(x)^3}\left(\left(4\cos(x)^2\ln\left(\frac{2}{\cos(x)+1}\right)-4\cos(x)^2\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)+\cos(x)^2-4\ln\left(\frac{2}{\cos(x)+1}\right)+4\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)+1\right)\sqrt{4}\sin(x)\left(\frac{\cos(x)^2}{\cos(x)^2-1}\right)^{3/2}\right)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-1+\csc(x)^2}\,dx$$

Optimal(type 3, 12 leaves, 3 steps):

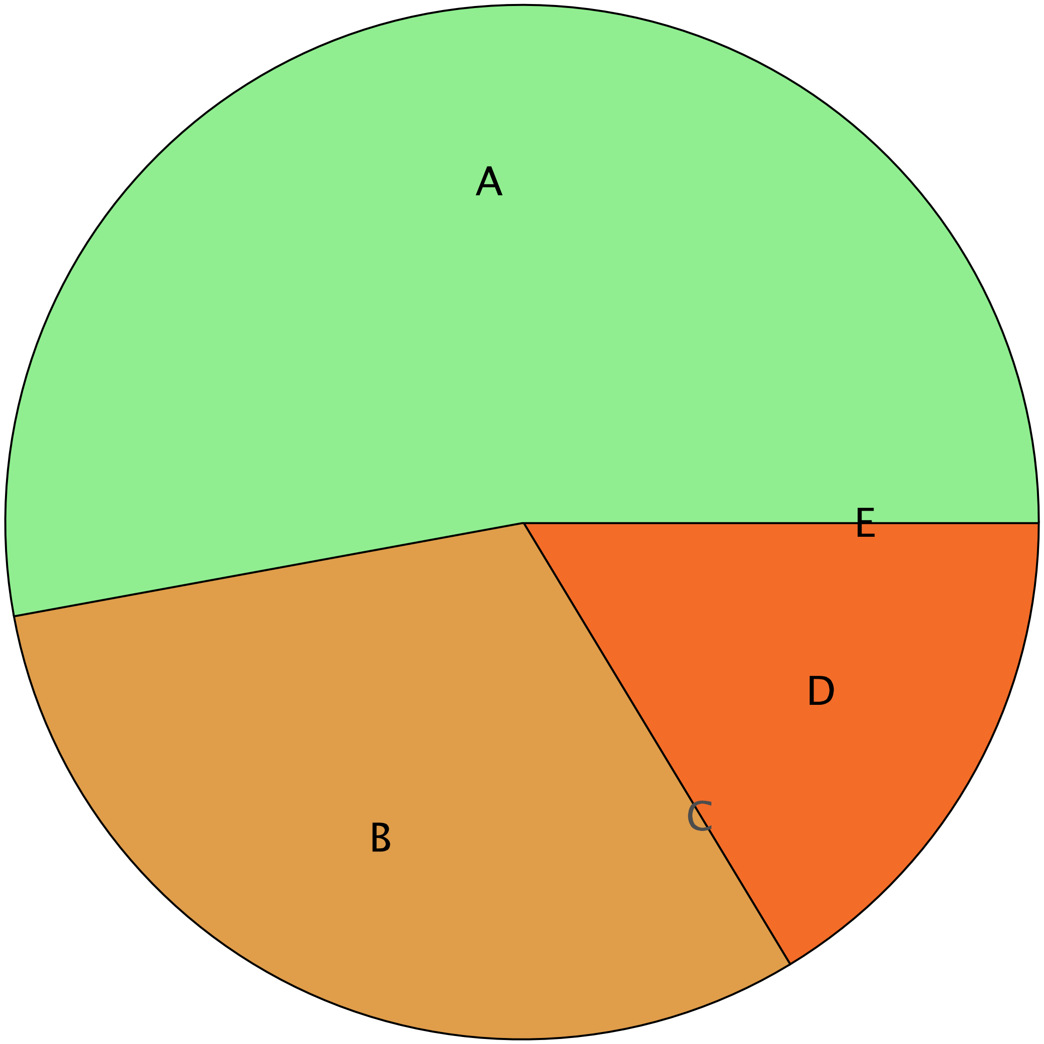
$$\ln(\sin(x))\sqrt{\cot(x)^2}\tan(x)$$

Result(type 3, 50 leaves):

$$-\frac{\sqrt{4}\left(\ln\left(\frac{2}{\cos(x)+1}\right)-\ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right)\right)\sin(x)\sqrt{-\frac{\cos(x)^2}{\cos(x)^2-1}}}{2\cos(x)}$$

Summary of Integration Test Results

104 integration problems



A - 55 optimal antiderivatives
B - 32 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 17 unable to integrate problems
E - 0 integration timeouts